(9) VKSU

Relation between Phas velocity (Wave velocity) and Group velocity

Consider two frequency Components have different Phase velocities

$\frac{\omega_1}{K_1}$ and $\frac{\omega_2}{K_2}$ So That $\frac{\omega_1}{K_1} \neq \frac{\omega_2}{K_2}$,

Then The velocity

$V_g = \frac{\omega_1 - \omega_2}{K_1 - K_2} = \frac{\Delta \omega}{\Delta K}$ will be different

from each of The Phase velocities.

In Such a Case The Superposition of the two waves will no longer remain Constant and hence The group Profile will change with time. Thus Such a medium in which The Phase velocity depends upon frequency is Called dispersive Medium. So The Variation of ω as a function of K defines a dispersion relation.

If The group of waves Consists of a no. of Component velocities very close to each other, hence we Can write as:

$$\frac{\Delta \omega}{\Delta K} = \frac{d\omega}{dK} = V_g \quad \text{——} \quad (1)$$

If The Phase velocity is defined by v Then.

$$v = \frac{\omega}{K} \quad \text{or} \quad \omega = Kv$$

$$\frac{d\omega}{dK} = v + K\frac{dv}{dK}$$

$$V_g = \frac{d\omega}{dK}$$

$$V_g = v + \frac{K}{dK} dv \quad \text{——} \ (2)$$

Also,

$$K = \frac{\omega}{v} = \frac{2\pi\omega}{v} = \frac{2\pi}{\lambda} \quad \text{——} \ (3)$$

Now differentiating equ^n (3) w.r.t. λ.

$$\frac{dK}{d\lambda} = -\frac{2\pi n}{\lambda^2} = -\frac{1}{\lambda} K$$

$$\frac{K}{dK} = -\frac{\lambda}{d\lambda}$$

Now substituting the above value of K/dK in equation (2), we get

$$V_g = v - \lambda \frac{dv}{d\lambda}$$

Thus the group velocity V_g can be $v - \lambda \frac{dv}{d\lambda}$, where v is the wave or phase velocity.
Now there are three cases arises.

(1) If $\frac{dv}{d\lambda}$ is a +ve quantity, in this case V_g (group velocity) is less than wave velocity.

② If $\frac{dv}{d\lambda} = 0$, in this group velocity will be equal to wave or Phase velocity. Thus we can say that if the phase velocity is same for all the component waves, the group velocity of the wave can be finding by their superposition is equal to wave or phase velocity.

③ If $\frac{dv}{d\lambda}$ is a –ve quantity, in this case group velocity is larger than the wave velocity.

Difference between Phase velocity and group velocity.

——— x ———→ x ———→

There are many differences in between phase velocity and group velocity. discussed below in table.

Table

Phase velocity :

1.) The Phase velocity is greater as compared to the group velocity in a normal medium.

2.) In Phase velocity, both Single waves as well as Superimposed waves are defined.

3) phase velocity can be defined as The velocity of the wave with higher frequency.

4.) Phase velocity deals with the velocity of a monochromatic wave.

Group velocity :

1.) The group velocity is greater as Compared to The Phase velocity in an anomalous medium.

2.) In group velocity, only Superimposed wave is defined.

3.) Group velocity Can be defined as The velocity of The wave with lower frequency.

4.) Group velocity deals with which the entire group of waves would travel.